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LETTER TO THE EDITOR

Topological and geometrical properties of DLA clusters

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Abstract. Three new exponents are calculated for diffusion limited aggregates in two dimensions via computer simulation. The $1/N=0$ limit gives $\zeta=1.52$, $\nu=0.91$ and $d_w=2.56$. These results confirm Martin's conjecture $D=\zeta/\nu$, and are consistent with previous work indicating that topological rather than geometrical properties of random fractals are responsible for variations in fractal dimension. Diffusion on DLA clusters is anomalous with an exponent that is consistent with the Alexander-Orbach conjecture and satisfies the Einstein relation.

Martin (1985) has recently attempted to refine the characterisation of random fractals by introducing two exponents, ζ and ν, which describe the topological and geometrical connectivities of a structure. From previous work on percolation clusters (Majid *et al* 1984), percolation backbones (Hong *et al* 1984) and lattice animals (Havlin *et al* 1984) it appears that the qualitative variations in structure can be traced to differences in ζ, whereas ν tends to have a value close to 0.90 (in two dimensions) for all of the above systems.

In this letter we present the determination of these exponents for the case of diffusion limited aggregation in two dimensions. These results imply that DLA follows the general pattern noted above—the geometrical exponent has a value of 0.91, which is roughly consistent with lattice animals, percolation clusters and percolation backbones. In addition we have also determined the exponent for anomalous diffusion (Gefen *et al* 1983); it appears that the Alexander-Orbach conjecture (Alexander and Orbach 1982) is satisfied for DLA in two dimensions.

All three exponents are determined by considering various walks on a lattice fractal. ζ and ν are determined by considering the set of minimal paths on a cluster; a minimal path is the shortest path between two points that visits only occupied lattice sites. ζ and ν are then defined by

$$\eta_l \sim l^{\zeta-1}$$

$$\langle r_l^2\rangle^{1/2} \sim l^{\nu}$$

where n_l is the number of minimal paths of length l, and $\langle r_l^2\rangle^{1/2}$ is the mean end-to-end distance of paths of length l.

The diffusion exponent d_w is determined by executing a large number of random walks on the cluster and measuring the mean end-to-end geometric distance of each walk. For random fractals this will vary with the number of steps taken in a power law fashion with exponent $1/d_w$:

$$\langle r^2(t)\rangle \sim t^{2/d_w}.$$

The clusters were generated on a 512×512 lattice according to the standard algorithm (Witten and Sander 1981). Clusters ranging in size from 100 particles to 5000 particles were generated, with the larger clusters of 1000, 2000, 3000 and 5000 particles being used for the extrapolation to infinite size. The smaller clusters were of interest in determining the effects of finite size upon fractal dimensionality. The simulations were carried out on relatively small minicomputers, which limited the maximum cluster size that could be generated in a reasonable time. This does not seem to be a problem; the 5000 particle clusters have values for the exponents that do not differ from the infinite size values by more than the numerical error.

The exponents ζ and ν were obtained by exactly enumerating all the minimal points emanating from a randomly chosen origin site; for each path the geometrical length r was calculated. The enumeration was carried out to paths of 30 steps. By averaging over several thousand starting sites the distributions n_l and $\langle r_l^2 \rangle$ were determined. A final averaging over ten clusters of each size completed the process. In addition, the density-density correlation function of each cluster was determined by the usual FFT procedure to determine the Hausdorff dimension D separately (Witten and Sander 1981).

Figure 1 shows the number of minimal paths against path length averaged over all clusters of size 5000. For small path lengths the deviation from power law behaviour is quite significant; consequently the exponent ζ was calculated using only the last 15 data points. In this case extending the length of the minimal paths will improve the accuracy of ζ, although we feel the present results are sufficient to illustrate the general trends.

The variation of geometrical distance with path length for the same data set is shown in figure 2. Here the power law behaviour is very evident. The reciprocal of the geometric exponent ν determines the scaling of resistance with distance if loops can be neglected, as is widely believed for DLA.

Figure 3 gives the anomalous diffusion results. These were obtained by executing 10 000 random walks of length 1 to 100, averaged over all clusters of a given size. The

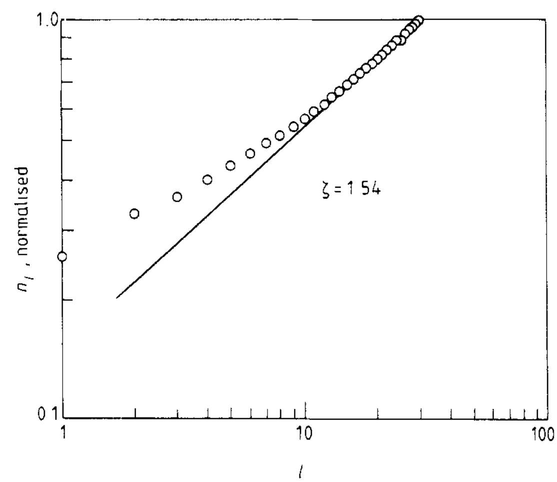

Figure 1. Number of minimal paths n_l against topological length l, averaged over all clusters of 5000 particles. n_l was arbitrarily normalised to a maximum value of 1. Power law behaviour occurs for lengths greater than about 10.

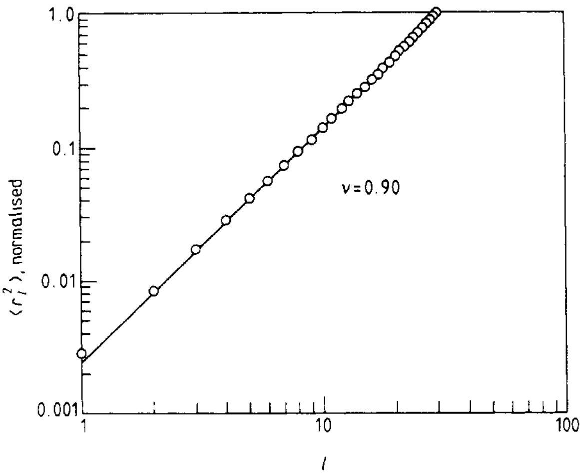

Figure 2. Mean square distance $\langle r_l^2 \rangle$ of minimal paths against topological distance l, averaged over all clusters of 5000 particles.

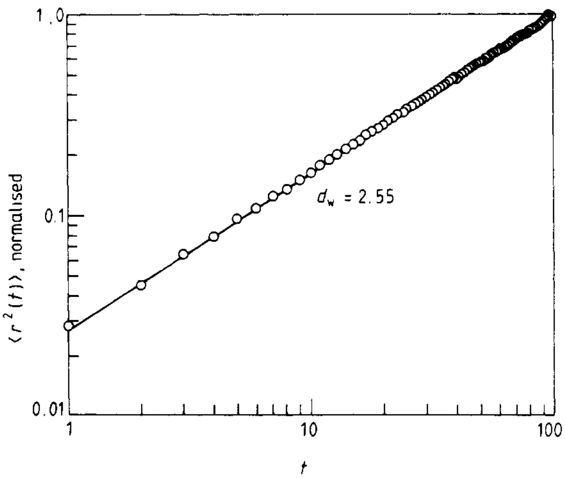

Figure 3. Mean square distance $\langle r^2(t) \rangle$ of random walks against number of steps t, averaged over all clusters of 5000 particles.

Einstein relation, which relates the DC conductivity to the carrier mobility, can be used to obtain a scaling relation between the random walk exponent d_w, the geometric exponent ν and the fractal dimension D (Gefen *et al* 1983):

$$d_w - D = \nu.$$

Our values for the infinite size limits of these exponents satisfy this relation.

The size dependence of the exponents is fairly significant. When the origin site is close to the boundary of the cluster the number of minimal paths found for that site is reduced, hence reducing the value of ζ. There should be no size dependence, however, for ν, since it is the result of end-to-end distance measurements on a set of minimal paths and does not depend on the number of such paths. The diffusion exponent d_w is expected to show some size dependence due to a reduction in geometric length for random walks originating near the boundary of a cluster. Figure 4 shows this behaviour quite clearly. By extrapolating to $1/N=0$ we find that the limiting values are: $\zeta=1.52\pm0.04$, $\nu=0.91\pm0.01$ and $d_w=2.56\pm0.02$. The ratio ζ/ν is indeed equal to $D=1.67$ as proposed by Martin (and stated in slightly different form by Havlin *et al*). Using these values we find the fracton dimension $d_s=2D/d_w=1.31$, which is consistent with the Alexander-Orbach conjecture $d_s=\frac{4}{3}$ within our errors, at least for $d=2$.

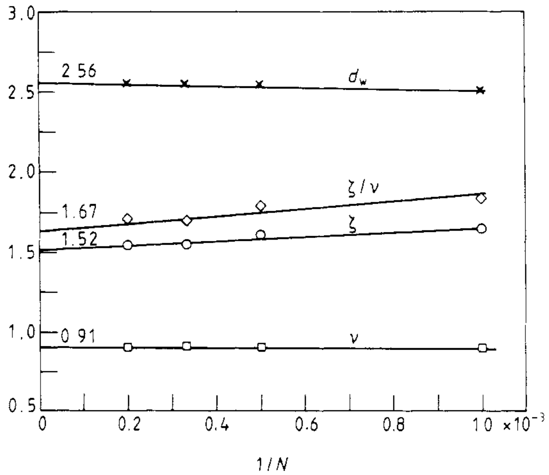

Figure 4. Size dependence of exponents. Cluster sizes are 1000, 2000, 3000 and 5000 particles. The $1/N=0$ limits satisfy all required scaling relations; the ratio ζ/ν is equal to the accepted value of D (1.67) for two-dimensional DLA.

Our result for ζ lies between percolation clusters and percolation backbones, as shown in table 1. While both backbones and DLA clusters have almost identical fractal dimensions, they are quite different structures qualitatively. It is this difference that is reflected in the values of the topological exponent.

Table 1. Values of exponents for DLA, percolation clusters and backbones, and lattice animals in two dimensions. The DLA values are from the present work: the percolation results are from Hong *et al* (1984) and Majid *et al* (1984). Values in parentheses are derived from other exponents.

	D	ζ	ν	d_w	d_s
DLA	1.67	1.52	0.91	2.56	(1.31)
Percolation clusters	1.89	1.63	(0.86)	(2.75)	(1.37)
Percolation backbones	1.66	1.44	(0.87)	(2.54)	(1.31)
Lattice animals	1.56	1.33	(0.85)	2.78	(1.12)

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References

Alexander S and Orbach R 1982 *J. Physique Lett.* **43** L625
Gefen Y, Aharony A and Alexander S 1983 *Phys. Rev. Lett.* **50** 77
Havlin S, Djordjevic Z, Majid I, Stanley H E and Weiss G H 1984 *Phys. Rev. Lett.* **53** 178
Hong D C, Havlin S, Hermann H J and Stanley H E 1984 *Preprint*
Majid I, Ben-Avraham D, Havlin S and Stanley H E 1984 *Phys. Rev.* B **30** 1626
Martin J E 1985 *J. Phys. A: Math. Gen.* **18** L207
Witten T A and Sander L M 1981 *Phys. Rev. Lett.* **51** 111